Propagation of Bessel Beam for Ground-to-Space Applications

CONFERENCE PAPER · JUNE 2015

DOWNLOADS 110
VIEWS 16

4 AUTHORS:

Iniabasi Ituen
National Space Research and Development A...
2 PUBLICATIONS 0 CITATIONS
SEE PROFILE

Phil Birch
University of Sussex
121 PUBLICATIONS 413 CITATIONS
SEE PROFILE

Chris R Chatwin
University of Sussex
370 PUBLICATIONS 1,334 CITATIONS
SEE PROFILE

Rupert C D Young
University of Sussex
235 PUBLICATIONS 927 CITATIONS
SEE PROFILE
Propagation of Bessel Beam for Ground-to-Space Applications

Iniabasi Ituen*, Philip Birch, Chris Chatwin, Rupert Young
Department of Engineering and Design, University of Sussex, Brighton, BN1 9QT, United Kingdom
*Author e-mail address: i.ituen@sussex.ac.uk

Abstract: We model the propagation of Gaussian and Bessel beams from ground through 22km altitude of atmospheric turbulence. We observe the Bessel beam has better performance based on RMS intensity error and the captured beam power.

OCIS codes: (010.1300) Atmospheric propagation; (000.1330) Atmospheric turbulence.

1. Introduction
Free Space Optics (FSO) for data transmission has so far largely focused on the propagation of Gaussian beams. However, Gaussian beams suffer from diffraction, causing the spread of the beam’s energy and so lowering the signal to noise ratio at the receiver. This paper investigates improvements to FSO by simulating the propagation of non-diffracting beams through unguided media. With the main impairments to FSO known to be diffraction and atmospheric turbulence, Durnin’s [1] idea of a non-diffracting self-healing Bessel beam could potentially mitigate these problems.

Bessel beams possess an intensity profile that is cylindrically symmetrical: a central core surrounded by a set of concentric rings. It has been shown that the central core of a Bessel beam is remarkably resistant to diffractive spreading compared to that of a Gaussian beam with a similar beam radius [2,3]. Bessel beams can be decomposed into an infinite set of plane wavefronts at different azimuths, but at a fixed inclination towards the direction of travel. When propagating, these wavefronts travel inwardly adding up to the energy of the central core [1,4,5]. This inward diffraction helps the on-axis intensity to remain constant as it propagates. Another effect of the inward diffraction is an attribute called self-healing: the beam is capable of recovering back its profile after being partially scattered by an obstruction. These properties make Bessel beams very promising for various applications such as observatory astronomy as well as terrestrial and satellite communications.

In this paper, we simulate the propagation of both Gaussian and Bessel beams from ground level, through atmospheric turbulence. Previously, Nelson et al [4] investigated the propagation of these beams over a short ground-to-ground range of 6.4km, with constant strength of turbulence $C_n^2$. In this paper we investigate the more difficult propagation problem from ground to space by considering the $C_n^2$ to be larger in the lower atmosphere but gradually weakening with altitude, based on some modifications to the Hufnagel-Valley model [6,7] over a considerably larger distance.

2. Background
A. Gaussian Beam
Most lasers produce beams with Gaussian profiles. The Gaussian beam is known to have a longitudinal profile with a waist, $w_0$ beyond which the beam begins to diverge at a constant angle, $\theta$. For a Gaussian beam of wavelength $\lambda$, the beam cross section (also called spot size) after being propagated to a distance $z$ is given by the equation

$$w(z) = w_0 \sqrt{1 + \left(\frac{z\lambda}{\pi w_0}\right)^2} \quad (1)$$

As the beam diverges down the path, the spot size doubles at a distance from the waist, $z_{2w} = \frac{\sqrt{3}\pi w_0}{\lambda}$. The beam divergence results in losses in the eventual beam power that reaches the receiver telescope.

B. Bessel Beam
An ideal $n^{th}$ order Bessel beam in terms of radial $r$, azimuthal $\phi$ and longitudinal $z$ components is given by [3]:

$$E(r, \phi, z) = A_0 \exp\left(i k_z z\right) J_n \left(k_r r\right) \exp\left(\pm i n \phi\right) \quad (2)$$

where $k_r$ and $k_z$ are the radial and axial wave vectors, respectively. The wave number $k = \sqrt{k_r^2 + k_z^2}$.

The Bessel beam in equation 2 is infinite in extent but in reality is truncated by some aperture function of radius R. This aperture results in the Bessel beam only being non-diffracting over a finite distance. The diffraction free attribute of the Bessel beam is limited to a certain range, $Z_{max}$ expressed as [1,3]:
\[ Z_{\text{max}} = \frac{R}{\tan \theta} = \frac{Rk}{k_r} \]  

Equation 3 shows that to propagate the Bessel beam a long distance, either \( R \) has to be large, making the telescope inconveniently large, or \( k_r \) must be made small. By limiting \( k_r \) such that the aperture function radius is equal to the Bessel function’s first root:

\[ R = \frac{2.405}{k_r} \]

we can determine the maximum possible propagation distance:

\[ Z_{\text{max}} = \frac{r^2k}{2.405} \]

This allows us to directly equate the maximum distance the beam can propagate with the radius of the transmitter telescope. The power contained within the first main unto the first zero with peak intensity \( I_0 \) is:

\[ P = \int_0^{|w_e|} I_0 \left( r_k r \right) dr = 2.405 F_3 \left( \frac{1}{2}, \frac{1}{2}, 1, \frac{3}{2}; -2.405^2 \right) = I_0 1.1386 \]

where \( F_3 \) is the generalised hypergeometric function.

C. Atmospheric Turbulence

It is well known that atmospheric turbulence is dominant in the lower atmosphere due to severe variations in the refractive index of air. Typical values for \( C_n^2 \) are known to range from 10\(^{-13} \) to 10\(^{-18} \) m\(^{-2/3} \). Andrews et al [8] made experimental contributions to the Hufnagel-Valley \( C_n^2 \) model [6,7]. Based on this, Andrew’s modification and by assuming \( M=6.5, h_i = h_o = 0 \) [8], we arrive at an expression for the \( C_n^2 \) in terms of the altitude, \( h \)

\[ C_n^2(h) = 6.5 \left[ 3.593 \times 10^{-3} \left( \frac{h}{10^5} \right)^{10} e^{(-h/1000)} + 2.7 \times 10^{-16} e^{(-h/1500)} \right] \]

From equation 7, we derive a maximum \( h \) of 22km above which the \( C_n^2(h) \) would be less than 10\(^{-18} \) m\(^{-2/3} \) and hence negligible.

3. Simulation

Both Gaussian beams and zeroth order Bessel beams were simulated and propagated through 100 phase screens representing layers of atmosphere with a varying structure parameter defined by equation 7. Fresnel diffraction was used to simulate the diffraction between the layers. For applications that go higher than the 22km, the beam was further propagated in vacuo since the effects of turbulence are assumed to be negligible in this region.

The wavelength of the beams was chosen as 0.5\( \mu \)m, the source and observation plane size was 2m x 2m with 1024x1024 pixels. The Bessel beam was masked to the size of its central core and compared to a Gaussian beam of 1/e\(^3 \) spot size. Different initial beam aperture radii where propagated ranging from 0.05 to 0.3m.

4. Results and Discussion

The results discussed here were obtained by averaging over 50 runs of the model. Figure 1 presents a plot of the beam wander: the drift from the initial centroid position (pixel position of maximum intensity) as it propagates through turbulence. The beam wander effect on both the Gaussian and Bessel beams is observed to be almost identical. Figure 2 displays the RMS intensity variation of the received beam after being diffracted as well as distorted by the turbulence. The Gaussian beam produces more intensity errors than the Bessel beam. Furthermore, we observe that the smaller the beam width the more the error, implying that larger telescope beam widths are more resistant to atmospheric turbulence as would be expected.

In figure 3, we demonstrate the captured power of the beam received by a receiving aperture equal in size to the transmitting one. The power considered here is normalized to that of 0.3m radius beam. For many of the beam sizes, the Bessel beam out-performs the Gaussian. It should be noted that due to space constraints, only metrics within the atmosphere are shown. Once the beams leave the atmosphere and is in vacuum, provided the propagation length is less than \( Z_{\text{max}} \), the Bessel beam continues to propagate in a non-diffracting manner and so outperforms the Gaussian beam.

5. Conclusion

We considered the propagation of Bessel and Gaussian beams from ground through 22km altitude of atmospheric turbulence which is gradually weakening with height. We examined the beam wander effect on the propagated beams and found that the centroid position of both beams is distorted due to turbulence by a similar amount. We have also investigated the RMS intensity variation of the received beam from the source beam after propagating...
through the turbulence. The Bessel beam proved to have less intensity variation than the Gaussian, implying better performance for the non-diffracting beam. Again, we demonstrated that, for high altitude applications, larger beam sizes from large telescopes are preferable as they are more resistant to turbulence. Finally, the Bessel beam remained more collimated as it propagated through the turbulence and hence more of its power could be captured by the receiver than for the Gaussian beam.

![Figure](image_url)

Figure 1, beam wonder. Centroid pixel position of the beam versus distance. Dot-Dash line are the Gaussian beams, solid line represents the Bessel beam. Both the Bessel and Gaussian beams suffer from almost identical amounts of beam wander. Beam radius are denoted by: Circle 0.05m, Plus 0.07m, Asterisk 0.1m, X 0.13m, Square 0.16m, Diamond 0.18m, Up triangle 0.21, Down triangle 0.244, Right triangle 0.27m, Left triangle 0.3m

![Figure](image_url)

Fig. 2. RMS Intensity Variation (legend as fig1).

Fig. 3. Normalised captured power (legend as fig1)

## 6. References