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Article (Accepted Version)

Regan, Donough, Anderson, Gemma Jayne, Hull, Matthew and Seery, David (2015) Constraining Galileon inflation. *Journal of Cosmology and Astroparticle Physics*, 2015 (02). ISSN 1475-7516

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Constraining Galileon Inflation

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Abstract. In this short paper, we present constraints on the Galileon inflationary model from the CMB bispectrum. We employ a principal-component analysis of the independent degrees of freedom constrained by data and apply this to the WMAP 9-year data to constrain the free parameters of the model. A simple Bayesian comparison establishes that support for the Galileon model from bispectrum data is at best weak.

1 Introduction

Recent microwave background data from the Planck satellite suggest that the pattern of density fluctuations in our universe is consistent with a canonical, single-field, slow-roll inflationary model [1]. To test for deviations from this paradigm we typically search for signatures in the the n -point functions of the microwave background anisotropies. At the time of writing, meaningful constraints have been obtained for the cases $n = 2$ and $n = 3$ —respectively, the power spectrum and the bispectrum, corresponding to spectral decompositions of the variance and skewness.

In this paper we focus on searches using the bispectrum, usually conducted by comparing fixed ‘templates’ to the data. This is useful in a discovery phase, where the relevant question is only whether evidence exists for the amplitude of *some* template to be inconsistent with zero. However, because templates do not accurately explore the range of shapes produced in a specific model, it would be more satisfactory to search for evidence for the model as a whole, rather than focusing on separate templates.

How should this be done? A given experiment measures each angular component of the bispectrum with varying signal-to-noise, depending on its instrumental characteristics. Therefore different experiments are sensitive to differing contributions to the bispectrum. For any chosen experiment, a typical model will predict contributions to which it is highly sensitive and others to which it is comparatively blind. We can expect to constrain only those parameters of a model which contribute to regions of sufficient sensitivity. Fitting our models to these regions simultaneously gives a balanced picture of the goodness-of-fit associated with the experiment. Fitting separate templates may not produce such a balanced picture if it fails to take all experimentally-sensitive regions into account.

Byun & Bean have used this approach to develop forecasts for a Planck-like experiment [2]. More recently, some of us applied similar reasoning to the WMAP 9-year dataset and a very general model described by the effective theory of single-field inflation [3]. This theory describes the most general pattern of fluctuations which can be realized in a Lorentz-invariant field theory, assuming Lorentz invariance to be spontaneously broken by a nearly de Sitter background. Because it can describe any adiabatic fluctuation with sufficiently smooth statistical properties it can be regarded as a weak prior—and, as for any prior, more stringent constraints can be obtained by strengthening it. One reason for doing so is to explore how the interpretation of the data changes as we vary our assumptions. Another is to study how the constraints improve when we commit to a particular model, rather than allowing for the most general range of possibilities.

In this paper we focus on a particular prior for the nonlinear stochastic properties of the inflationary density perturbation—that it was generated during an era of ‘Galileon’ inflation [4]; see also Refs. [5–9]. Galileons are scalar fields with highly constrained self-interactions which contain higher-order time derivatives. These cancel in the equations of motion [10], yielding stable second-order field equations. When quantized this implies that the theory is ghost-free, and therefore maintains unitarity and stability.

These stability properties are preserved by quantum fluctuations around flat Minkowski space. At present it is apparently unclear whether the ghost-free theory can arise as an effective description of a theory with an ultraviolet completion [11], which would require the special Galileon self-interactions to be unaccompanied by other higher-derivative operators which would generate a ghost. It is also unknown whether the ghost-free property survives on a cosmological background or in the field created by a heavy source. But if these possible complications can be evaded and a field with Galileon-like interactions were dynamically important during inflation, then it is possible that their special nature could leave interesting signatures in the stochastic properties of the density perturbation [12–15].

Our principal result is a constraint on the importance of the Galileon self-interactions which would generate these signatures. For this purpose the Galileon model is particularly interesting because it allows just three 3-body interactions compared to the eleven allowed by the unconstrained effective field theory. In Ref. [3] we argued that the WMAP 9-year dataset is sensitive to three or (at most) four characteristic contributions to the bispectrum. Unless we are unlucky and the Galileon 3-body interactions contribute to these regions in a degenerate way, we can expect to obtain constraints on all three couplings.

More generally, the operators of the Galileon Lagrangian form a subset of the class of single-field effective theories of inflation. The symmetries of the Galileon model impose a relationship between the coefficients of the possible Lagrangian terms. In this paper we exploit this relationship to present the strongest possible constraints on the Galileon paradigm.

Notation.—In §2 we briefly describe the action for the Galileon inflationary model up to third order, and in §3 we use it to derive the bispectrum. In §4 we describe the procedure used to obtain our constraints. This is a summary of the approach developed in Ref. [3]. We obtain constraints on the couplings in the Lagrangian for the cases of one, two or three (the most general possibility) independent third-order couplings. Finally, in §5 we carry out a

Bayesian model comparison for the various incarnations of the Galileon model. We give our conclusions in §6.

2 Overview of Galileon inflation

The original Galileon model constructed by Nicolis et al. was based on a field-space generalization of the Galilean shift-symmetry of classical mechanics, $\phi \rightarrow \delta_g \phi = \phi + b_\mu x^\mu + c$ where x^μ is a spacetime coordinate and b_μ and c are constants [18]. Nicolis et al. worked in flat spacetime and constructed four operators, labelled \mathcal{L}_i for $2 \leq i \leq 5$, which yielded an action satisfying this symmetry¹ and produced second-order equations of motion. On a curved background it is not possible to retain both properties [19–21]. Insisting on second-order equations of motion and accepting a break of the shift symmetry proportional to the background curvature yields the ‘covariantized’ formulation, with action

$$S \supseteq \int d^4x \sqrt{-g} [c_2 \mathcal{L}_2 + c_3 \mathcal{L}_3 + c_4 \mathcal{L}_4 + c_5 \mathcal{L}_5], \quad (2.1)$$

and \mathcal{L}_i defined by

$$\mathcal{L}_2 = \frac{1}{2} (\nabla \phi)^2 \quad (2.2a)$$

$$\mathcal{L}_3 = \frac{1}{\Lambda^3} \square \phi (\nabla \phi)^2 \quad (2.2b)$$

$$\mathcal{L}_4 = \frac{(\nabla \phi)^2}{\Lambda^6} \left[(\square \phi)^2 - \nabla_\mu \nabla_\nu \phi \nabla^\mu \nabla^\nu \phi - \frac{R}{4} (\nabla \phi)^2 \right] \quad (2.2c)$$

$$\mathcal{L}_5 = \frac{(\nabla \phi)^2}{\Lambda^9} \left[(\square \phi)^3 - 3 \square \phi \nabla_\mu \nabla_\nu \phi \nabla^\mu \nabla^\nu \phi + 2 \nabla_\mu \nabla_\nu \phi \nabla^\nu \nabla^\rho \phi \nabla_\rho \nabla^\mu \phi - 6 G_{\mu\nu} \nabla^\mu \nabla^\rho \phi \nabla^\nu \phi \nabla_\rho \phi \right]. \quad (2.2d)$$

Here, $G_{\mu\nu}$ is the Einstein tensor, R denotes the scalar curvature of the background and Λ is a mass scale at which the higher-order operators \mathcal{L}_3 , \mathcal{L}_4 and \mathcal{L}_5 become comparable to the Gaussian term \mathcal{L}_2 .

These are not the only nonlinear operators which yield an action invariant under the shift symmetry in flat space—for example, any function of $\square \phi$ is automatically invariant—but they are the only combinations which produce second-order equations of motion. Consistency of the model requires that unwanted combinations such as $(\square \phi)^2/M^2$ (which would generally indicate the presence of a ghost at the scale M) are not generated by renormalization-group running for any M at which we wish to trust the predictions of the effective theory.² This does not happen in the vacuum, where the ϕ fluctuations are massless and do not generate renormalization-group evolution. It is not yet known whether a ghost will appear for non-vacuum field configurations [11].

¹The \mathcal{L}_i themselves need not be invariant, provided that the transformation shifts them only by a total derivative which vanishes on integration.

²It is inconsequential if a ghost is generated at scales which are not intended to be described by the effective Lagrangian: this happens generically in any effective field theory. To understand whether the putative ghost really exists in the spectrum we would need details of the ultraviolet completion.

In this paper we assume that the action (2.1) can be used to describe fluctuations on a quasi-de Sitter background representing an inflationary phase, and that there is a regime for which quantum effects do not cause ghost modes to become excited. Working in the ‘decoupling’ limit where gravitational degrees of freedom can be ignored, it was shown in Ref. [4] that the action for small fluctuations can be written

$$S \supseteq \int d^4x a^3 \left[\alpha \left(\dot{\pi}^2 - \frac{c_s^2}{a^2} (\partial\pi)^2 \right) + g_1 \dot{\pi}^3 + \frac{g_3}{a^2} \dot{\pi} (\partial\pi)^2 + \frac{g_4}{a^4} (\partial\pi)^2 \partial^2\pi \right]. \quad (2.3)$$

The decoupling limit applies when the higher-order terms \mathcal{L}_3 , \mathcal{L}_4 and \mathcal{L}_5 are relevant, making Galileon self-interactions stronger than gravitational interactions. But if pushed too far these self-interactions require a rapidly evolving field configuration which risks spoiling the de Sitter background. A complete understanding of what happens in this regime would require an analysis of the associated ‘cosmological’ Vainshtein effect, which has not yet been carried out. We assume that these complications can be evaded by having the higher-order terms sufficiently relevant that they dominate gravitational corrections, but not so relevant that they destabilize the inflationary era. The coefficients α , c_s , g_2 , g_3 and g_4 are given by various combinations of c_i , H , $\dot{\phi}$ and the nonlinear scale Λ . For precise formulae, or further discussion of the role of the nonlinear terms, we refer to Burrage et al. [4]. On superhorizon scales the curvature perturbation is given by $\zeta = H\pi$, and is conserved.

Constraints on this model take the form of limits on the parameters c_i . Some limits exist based on short-distance gravitational effects in the late universe; see, for example, Ref. [22]. There is no particular requirement for the Galileon-like fields relevant during the early and late universe to have the same identity (although this may be the case in certain models), so these limits need not apply during inflation. In this paper we obtain limits on the c_i from the bispectrum of the inflationary density perturbation without making use of any late-universe data.

Note that the fluctuations generated in certain k -inflation and Horndeski models may be controlled by the same action [27]. Therefore our results can equally be interpreted as constraints on these models, although we do not identify them explicitly. Within this large class of theories, Galileons are algebraically special in that they require only three independent cubic operators, as in Eq. (2.3). A generic k -inflation or Horndeski model may require up to four independent operators [13, 27].

3 Bispectrum Shapes

The bispectrum, B_ζ , is defined by the three-point correlation function of the curvature perturbation,

$$\langle \zeta(\mathbf{k}_1) \zeta(\mathbf{k}_2) \zeta(\mathbf{k}_3) \rangle = (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) B_\zeta(k_1, k_2, k_3). \quad (3.1)$$

Wavefunctions.—A complication of the most general models studied in Ref. [3] is that fourth-derivative operators can appear in the quadratic term, leading to a very complex form for the elementary functions. While a solution can be found in closed form, the subsequent vertex integrations appearing in the Feynmann rules cannot be performed analytically. The

Galileon fluctuation Lagrangian (2.1) belongs to the class of models considered Ref. [3] but does not possess the problematic fourth-derivative operators. Neglecting slow-roll corrections the elementary wavefunction is

$$u(k, \tau) = \frac{iH}{2\sqrt{\alpha}} \frac{1}{(kc_s)^{3/2}} (1 - ikc_s\tau) e^{ikc_s\tau}. \quad (3.2)$$

On superhorizon scales where $|kc_s\tau| \ll 1$ the power spectrum can be written

$$P_\zeta(k) = \frac{H^4}{4\alpha c_s^3} \frac{1}{k^3}. \quad (3.3)$$

Momentum dependence.—The necessary calculations were described by Mizuno & Koyama [9], and given to next-order in slow-roll parameters in Refs. [4, 13]. Similar calculations were performed by Kobayashi, Yamaguchi and Yokoyama [7]. There is one contribution to the bispectrum from each cubic operator in (2.3), which we write in the form

$$B_\zeta(k_1, k_2, k_3) = \frac{3}{5} \sum_{\alpha=1}^3 \lambda_\alpha B_\alpha(k_1, k_2, k_3). \quad (3.4)$$

The B_α are normalized so that $B_\alpha(k, k, k)/6P_\zeta(k)^2 = 1$ at the equilateral point. In terms of the couplings g_α in the fluctuation Lagrangian these means that that λ_α correspond to

$$\lambda_1 = \frac{5}{81} \frac{g_1}{\alpha} \quad \lambda_2 = -\frac{85}{324} \frac{g_3}{c_s^2 \alpha} \quad \lambda_3 = -\frac{65}{162} \frac{g_4 H}{c_s^4 \alpha}. \quad (3.5)$$

Focusing only on the momentum dependence (prefactors can be inferred from the normalization convention if required), the bispectra can be written

$$B_1(k_1, k_2, k_3) \sim \frac{1}{\prod_i k_i} \frac{1}{k_t^3} \quad (3.6a)$$

$$B_2(k_1, k_2, k_3) \sim \frac{1}{\prod_i k_i^3} k_1^2 (\mathbf{k}_2 \cdot \mathbf{k}_3) \left(\frac{1}{k_t} + \frac{k_2 + k_3}{k_t^2} + \frac{2k_2 k_3}{k_t^3} \right) + 1 \rightarrow 2 + 1 \rightarrow 3 \quad (3.6b)$$

$$B_3(k_1, k_2, k_3) \sim \frac{1}{\prod_i k_i^3} k_1^2 (\mathbf{k}_2 \cdot \mathbf{k}_3) \left(\frac{1}{k_t} + \frac{K^2}{k_t^3} + \frac{3k_1 k_2 k_3}{k_t^4} \right) + 1 \rightarrow 2 + 1 \rightarrow 3, \quad (3.6c)$$

where $k_t = k_1 + k_2 + k_3$, and $K^2 = k_1 k_2 + k_1 k_3 + k_2 k_3$.

4 Estimating Galileon Parameters

We now aim to estimate the λ_α using CMB data. Given a model, and therefore knowledge of the parameters α and c_s , this enables the g_α to be determined. Given knowledge of the background field configuration this enables constraints to be placed on the c_i .

Estimation methodology.—Our methodology for estimating the λ_α was described in Ref. [3]. Following Fergusson, Liguori & Shellard we write each of (3.6a)–(3.6c) as a sum over some

basis B_n , giving $B_\alpha = \sum_n \alpha_n^n B_n$ [23]. We extract multipole coefficients for the temperature anisotropy ΔT using $\Delta T(\hat{\mathbf{n}})/T = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{\mathbf{n}})$ and define the angular bispectrum $b_{\ell_1 \ell_2 \ell_3}$ to satisfy $\langle a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3} \rangle = b_{\ell_1 \ell_2 \ell_3} \mathcal{G}_{m_1 m_2 m_3}^{\ell_1 \ell_2 \ell_3}$, where $\mathcal{G}_{m_1 m_2 m_3}^{\ell_1 \ell_2 \ell_3}$ is the Gaunt integral. After using the primordial perturbation ζ to seed fluctuations in the radiation era, and accounting for radiative transfer and projection onto the sky, each B_n will yield some angular bispectrum $b_{\ell_1 \ell_2 \ell_3}^n$. We introduce a further basis $b_{\ell_1 \ell_2 \ell_3}^A$ and write $b_{\ell_1 \ell_2 \ell_3}^n = \Gamma_A^n b_{\ell_1 \ell_2 \ell_3}^A$. Then, given a choice λ_α , it follows that the observable angular bispectrum can be written

$$b_{\ell_1 \ell_2 \ell_3} = \sum_A \beta_A b_{\ell_1 \ell_2 \ell_3}^A, \quad (4.1)$$

where the coefficients β_A are defined by

$$\beta_A \equiv \lambda_\alpha \alpha_n^\alpha \Gamma_A^n. \quad (4.2)$$

The advantage of this basis decomposition is that the transfer matrix Γ_A^n can be computed relatively easily [24]. It encodes details of the cosmology, together with the processes of radiative transfer which connect primordial times to observation.

A given microwave background experiment makes measurements of the β_A associated with our last-scattering surface. We write these estimates $\hat{\beta}_A$. Assuming Gaussian experimental errors, and given our prior, the likelihood of an experiment returning some particular set of values can be written

$$\mathcal{L}(\hat{\beta}_A | \lambda_\alpha) = \frac{1}{\sqrt{2\pi \det \hat{\mathcal{C}}}} \exp\left(-\frac{1}{2} \sum_{A,B} (\hat{\mathcal{C}}^{-1})^{AB} \Delta \hat{\beta}_A \Delta \hat{\beta}_B\right), \quad (4.3)$$

where $\Delta \hat{\beta}_A \equiv \hat{\beta}_A - \beta_A$ and the covariance matrix is defined by $C_{AB} = \langle \Delta \hat{\beta}_A \Delta \hat{\beta}_B \rangle$. We estimate it using Gaussian simulations, accounting for realistic WMAP beam and noise properties and the effects of masking. For all quantitative details we refer to Ref. [3].

Eq. (4.3) yields a maximum likelihood estimator for each parameter λ_α corresponding to

$$\hat{\lambda}_\alpha = \sum_\beta \hat{b}^\beta (\hat{\mathcal{F}}^{-1})_{\beta\alpha}, \quad (4.4)$$

where $\hat{b}^\alpha \equiv \sum_{A,B,n} \hat{\beta}_A (\hat{\mathcal{C}}^{-1})^{AB} \alpha_n^\alpha \Gamma_B^n$ and the Fisher matrix $\hat{\mathcal{F}}$ satisfies

$$\hat{\mathcal{F}}^{\alpha\beta} = \sum_{A,B,m,n} \Gamma_A^m \alpha_m^\alpha (\hat{\mathcal{C}}^{-1})^{AB} \alpha_n^\beta \Gamma_B^n. \quad (4.5)$$

Application to 9-year WMAP data.—We use the WMAP 9-year dataset to estimate the amplitudes $\hat{\beta}_A$ [25, 26], and use these to constrain subcases of the fluctuation Lagrangian (2.3). The most general case includes all three operators g_1 , g_3 and g_4 and yields the weakest constraints. This would be expected where correlations among the operators exist in the regions to which WMAP is most sensitive. In this case rather more is true: Renaux-Petel pointed out [27, 28] that there is an approximate degeneracy (spoilt by boundary terms

which become irrelevant at late times) which allows the g_4 contribution to be absorbed into renormalizations of the other couplings,

$$g_1 \rightarrow g'_1 = g_1 + g_4 H/c_s^4, \quad g_3 \rightarrow g'_3 = g_3 + 2g_4 H/c_s^2. \quad (4.6)$$

We can leave g_4 in the analysis, accounting for the correlation in shape, or eliminate g_4 using (4.6) at the outset. In what follows we will give constraints for both choices. Finally, we consider the most restrictive subcase in which only one parameter is allowed to be nonzero. This corresponds most directly with the standard approach of fitting individual templates to the data. It gives optimistic constraints unless we are prepared to commit to a scenario in which two operators are subdominant compared to the third.

Case 1: General scenario (three free parameters).—Using the relationship between the parameters λ_α and the coefficients of the Lagrangian given in Eq. (3.5), we find

Variable	Estimate
\hat{g}_1/α	$(4.21 \pm 3.96) \times 10^5$
$\hat{g}_3/c_s^2\alpha$	$(4.18 \pm 4.03) \times 10^5$
$\hat{g}_4 H/c_s^4\alpha$	$(-2.09 \pm 2.04) \times 10^5$

The quoted uncertainties represent 1σ errors bars, with marginalization over the other two parameters. Each constraint is consistent with zero to within 1σ . Note that the uncertainties are rather large, due to exploration of the entire parameter space.

Case 2: Two free parameters.—It was explained above that the three-parameter case is perhaps too pessimistic, because of correlations between the bispectra produced by the three cubic operators. In this section we obtain constraints on the subcase where two couplings are allowed to vary with the third fixed at zero. In the case where g_4 is held fixed Eq. (4.6) can be used to map these constraints to the Lagrangian obtained by elimination of the g_4 term. The results are

Fixed parameter	Variable	Estimate
g_1/α	$\hat{g}_3/c_s^2\alpha$	$-11,000 \pm 6,770$
	$\hat{g}_4 H/c_s^4\alpha$	$7,760 \pm 4,870$
$g_3/c_s^2\alpha$	\hat{g}_1/α	$11,000 \pm 6,660$
	$\hat{g}_4 H/c_s^4\alpha$	$3,380 \pm 2,240$
$g_4 H/c_s^4\alpha$	\hat{g}_1/α	$15,300 \pm 9,470$
	$\hat{g}_3/c_s^2\alpha$	$2,870 \pm 1,900$

Each constraint is consistent with zero to within $\sim 1.5\sigma$, matching our expectations from constraints on individual templates using the 9-year WMAP dataset [26]. Similar results have been reported from the Planck [1] 2013 data release. The formalism used here ensures

that the entire available parameter space is explored, rather than inferring these constraints from the overlap with a selection of templates.

Case 3: One free parameter.—Finally, we consider the constraints where only one parameter is allowed to vary. We obtain

$$\hat{g}_1/\alpha = 1120 \pm 1280, \quad \hat{g}_3/c_s^2\alpha = -260 \pm 390, \quad \hat{g}_4H/c_s^4\alpha = -160 \pm 280. \quad (4.7)$$

We may also obtain a constraint from the amplitude of the power spectrum, which gives $H^4/(\alpha c_s^3) = (190 \pm 8) \times 10^{-9}$ at the pivot scale $k = 0.002\text{Mpc}^{-1}$. Together this gives 4 constraints for 6 parameters, $\{H, \alpha, c_s, g_2, g_3, g_4\}$. Breaking the degeneracy would require constraints on the scalar tilt, n_s or the tensor to scalar ratio, r .

5 Bayesian Model Comparison

While our results in the previous section are useful in determining best-fit values for the parameters, we wish also to perform a model comparison. One method with which to quantify to evidence for or against a model is through calculation of the ‘Bayes’ factor. In Ref. [3] this was first applied to the comparison of non-Gaussian models. We briefly recapitulate the description here.

Given a data set D and a pair of models M_1 and M_2 , with respective parameter sets $\{\lambda_1\}$ and $\{\lambda_2\}$, the Bayes factor is defined as the ratio of the likelihoods of the respective models,

$$K_{12} = \frac{P(D|M_1)}{P(D|M_2)} = \frac{\int P(D|\{\lambda_1\}, M_1)P(\{\lambda_1\}|M_1) d\lambda_1}{\int P(D|\{\lambda_2\}, M_2)P(\{\lambda_2\}|M_2) d\lambda_2}. \quad (5.1)$$

It should be noted that the integrals here are over the entire parameter space of each model. These may be of different dimensionalities. The prior probabilities $P(\{\lambda_i\}|M_i)$ represent the probability that a particular parameter choice occurs. To determine our priors we use the requirement that the bispectrum generated by each operator must not dominate the power spectrum, and therefore that each parameter is constrained by $|\lambda_i| \lesssim 10^4$. However, given that we have no reason³ to prefer any scale we choose a Jeffries prior with $P(\lambda_i) \propto 1/|\lambda_i|$, with $|\lambda_i| \in [1, 10^4]$. The cutoff is chosen to avoid a divergence at $\lambda_i = 0$, and our results show little dependence on its precise value. The Bayes factor does not become independent of the prior, so to study its dependence for different choices we also compute values for a flat prior in the range $[-1, 1]$. The probability $P(D|\{\lambda_1\}, M_1)$ represents the likelihood for a particular choice of parameters $\{\lambda_i\}$ and may be computed using Eq. (4.3).

We interpret our results using the Kass & Raftery scale [29]. In this scheme, $|\ln K|$ in the range $(0, 1)$ is ‘indecisive’, in the range $(1, 3)$ represents ‘evidence in favour of M_1 ’, in the range $(3, 5)$ represents ‘strong evidence in favour of M_1 ’, and larger values are ‘decisive’. A similar scale applies to K^{-1} with M_2 substituted for M_1 .

³We cannot use constraints from CMB experiments to choose our prior, because we are using the CMB as our dataset.

Results

- Comparing the Gaussian model (i.e. with all $\lambda_i = 0$) with the case of just one non-zero parameter, we find the Bayes factor is given by $\ln K \approx 0.7$, such that the data is indecisive in distinguishing these scenarios.
- Next we compare the Gaussian model and the case of two free parameters, with the result that $\ln K \approx 1.3$. This indicates (weak) evidence against the Galileon model with two free parameters, and is mainly due to an Ockham penalty which disfavours addition of extra parameters without sufficient support from the data. Adding a further parameter and comparing the Gaussian model to the most general Galileon model with three free parameters gives a Bayes factor $\ln K \approx 2$. In this case there is an even stronger preference for the simpler description.
- Comparing the single free parameter case with the two parameter and three parameter cases giving $\ln K \approx 0.6$ and $\ln K \approx 1.3$, respectively. The data is indecisive in the former case, but shows preference for the single parameter case in the latter.

In summary, the data shows little power to discriminate between the Gaussian model and a Galileon model with one extra free parameter. However, for models with two or more extra parameters the WMAP 9-year data exhibits a weak preference for the simpler description.

6 Conclusions

In this paper we have utilised the formalism developed in Ref. [3] to constrain the Galileon inflationary model using the bispectrum. Our constraints show that the couplings of the cubic terms in the fluctuation Lagrangian are consistent with zero to within 1.5σ . We have separately considered the cases of one, two and three free parameters in the fluctuation Lagrangian.

The formalism can be used to carry out a Bayesian model comparison. This establishes that the data weakly disfavours models requiring two extra free parameters, but is inconclusive between a Gaussian model and the case of a Galileon model with a single extra coupling. It is possible that carrying out the analysis using Planck data [1] may lead to a stronger result.

Acknowledgements

DR and DS acknowledge support from the Science and Technology Facilities Council [grant number ST/L000652/1]. GA is supported by STFC grant ST/L000652/1. MH is supported by an STFC research studentship. DS also acknowledges support from the Leverhulme Trust. Some numerical presented in this paper were obtained using the COSMOS supercomputer, which is funded by STFC, HEFCE and SGI. Other numerical computations were carried out on the Sciama High Performance Compute (HPC) cluster which is supported by the ICG, SEPNet and the University of Portsmouth. The research leading to these results has received funding from the European Research Council under the European Union's Seventh Framework Programme (FP/2007–2013) / ERC Grant Agreement No. [308082].

References

- [1] **Planck Collaboration** Collaboration, P. Ade *et al.*, *Planck 2013 Results. XXIV. Constraints on primordial non-Gaussianity*, [arXiv:1303.5084](#).
- [2] J. Byun and R. Bean, *Non-Gaussian Shape Recognition*, *JCAP* **1309** (2013) 026, [[arXiv:1303.3050](#)].
- [3] G. J. Anderson, D. Regan, and D. Seery, *Optimal bispectrum constraints on single-field models of inflation*, *JCAP* **1407** (2014) 017, [[arXiv:1403.3403](#)].
- [4] C. Burrage, C. de Rham, D. Seery, and A. J. Tolley, *Galileon inflation*, *JCAP* **1101** (2011) 014, [[arXiv:1009.2497](#)].
- [5] A. De Felice and S. Tsujikawa, *Primordial non-Gaussianities in general modified gravitational models of inflation*, *JCAP* **1104** (2011) 029, [[arXiv:1103.1172](#)].
- [6] T. Kobayashi, M. Yamaguchi, and J. Yokoyama, *G-inflation: Inflation driven by the Galileon field*, *Phys.Rev.Lett.* **105** (2010) 231302, [[arXiv:1008.0603](#)].
- [7] T. Kobayashi, M. Yamaguchi, and J. Yokoyama, *Primordial non-Gaussianity from G-inflation*, *Phys.Rev.* **D83** (2011) 103524, [[arXiv:1103.1740](#)].
- [8] T. Kobayashi, M. Yamaguchi, and J. Yokoyama, *Generalized G-inflation: Inflation with the most general second-order field equations*, *Prog.Theor.Phys.* **126** (2011) 511–529, [[arXiv:1105.5723](#)].
- [9] S. Mizuno and K. Koyama, *Primordial non-Gaussianity from the DBI Galileons*, *Phys.Rev.* **D82** (2010) 103518, [[arXiv:1009.0677](#)].
- [10] A. Nicolis, R. Rattazzi, and E. Trincherini, *The Galileon as a local modification of gravity*, *Phys.Rev.* **D79** (2009) 064036, [[arXiv:0811.2197](#)].
- [11] N. Kaloper, A. Padilla, P. Saffin, and D. Stefanyszyn, *Unitarity and the Vainshtein Mechanism*, [arXiv:1409.3243](#).
- [12] P. Creminelli, G. D’Amico, M. Musso, J. Norena, and E. Trincherini, *Galilean symmetry in the effective theory of inflation: new shapes of non-Gaussianity*, *JCAP* **1102** (2011) 006, [[arXiv:1011.3004](#)].
- [13] C. Burrage, R. H. Ribeiro, and D. Seery, *Large slow-roll corrections to the bispectrum of noncanonical inflation*, *JCAP* **1107** (2011) 032, [[arXiv:1103.4126](#)].
- [14] R. H. Ribeiro and D. Seery, *Decoding the bispectrum of single-field inflation*, *JCAP* **1110** (2011) 027, [[arXiv:1108.3839](#)].
- [15] G. Leon and E. N. Saridakis, *Dynamical analysis of generalized Galileon cosmology*, *JCAP* **1303** (2013) 025, [[arXiv:1211.3088](#)].
- [16] G. W. Horndeski, *Second-Order Scalar-Tensor Field Equations in a Four-Dimensional Space*, *International Journal of Theoretical Physics* **10** (Sept., 1974) 363–384.
- [17] C. Deffayet, X. Gao, D. Steer, and G. Zahariade, *From k-essence to generalised Galileons*, *Phys.Rev.* **D84** (2011) 064039, [[arXiv:1103.3260](#)].
- [18] A. Nicolis and R. Rattazzi, *Classical and quantum consistency of the DGP model*, *JHEP* **0406** (2004) 059, [[hep-th/0404159](#)].
- [19] C. Deffayet, G. Esposito-Farese, and A. Vikman, *Covariant Galileon*, *Phys.Rev.* **D79** (2009) 084003, [[arXiv:0901.1314](#)].

- [20] C. Deffayet, S. Deser, and G. Esposito-Farese, *Generalized Galileons: All scalar models whose curved background extensions maintain second-order field equations and stress-tensors*, *Phys.Rev.* **D80** (2009) 064015, [[arXiv:0906.1967](#)].
- [21] C. de Rham and A. J. Tolley, *DBI and the Galileon reunited*, *JCAP* **1005** (2010) 015, [[arXiv:1003.5917](#)].
- [22] C. Burrage and D. Seery, *Revisiting fifth forces in the Galileon model*, *JCAP* **1008** (2010) 011, [[arXiv:1005.1927](#)].
- [23] J. Fergusson, M. Liguori, and E. Shellard, *General CMB and Primordial Bispectrum Estimation I: Mode Expansion, Map-Making and Measures of f_{NL}* , *Phys.Rev.* **D82** (2010) 023502, [[arXiv:0912.5516](#)].
- [24] J. Fergusson, D. Regan, and E. Shellard, *Optimal Trispectrum Estimators and WMAP Constraints*, [[arXiv:1012.6039](#)].
- [25] **WMAP Collaboration** Collaboration, G. Hinshaw *et al.*, *Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Cosmological Parameter Results*, [[arXiv:1212.5226](#)].
- [26] **WMAP** Collaboration, C. Bennett *et al.*, *Nine-Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Final Maps and Results*, *Astrophys.J.Suppl.* **208** (2013) 20, [[arXiv:1212.5225](#)].
- [27] S. Renaux-Petel, *On the redundancy of operators and the bispectrum in the most general second-order scalar-tensor theory*, *JCAP* **1202** (2012) 020, [[arXiv:1107.5020](#)].
- [28] S. Renaux-Petel, *DBI Galileon in the Effective Field Theory of Inflation: Orthogonal non-Gaussianities and constraints from the Trispectrum*, *JCAP* **1308** (2013) 017, [[arXiv:1303.2618](#)].
- [29] R. E. Kass and A. E. Raftery, *Bayes factors*, *Journal of the American Statistical Association* **90** (1995), no. 430 773–795, [<http://amstat.tandfonline.com/doi/pdf/10.1080/01621459.1995.10476572>].