Asymptotic Safety Guaranteed in Supersymmetry

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We explain how asymptotic safety arises in four-dimensional supersymmetric gauge theories. We provide asymptotically safe supersymmetric gauge theories together with their superconformal fixed points, R charges, phase diagrams, and UV-IR connecting trajectories. Strict perturbative control is achieved in a Veneziano limit. Consistency with unitarity and the a theorem is established. We find that supersymmetry enhances the predictivity of asymptotically safe theories.

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Introduction.—The discovery of asymptotic freedom for non-Abelian gauge theories in 1973 has initiated a new era in particle physics [1,2]. Asymptotic freedom explains why certain types of quantum field theories such as the strong and weak sector of the standard model can be truly fundamental and predictive up to the highest energies. It implies that interactions are switched off asymptotically and theories become free. Asymptotic freedom constitutes a cornerstone in the standard model of particle physics and continues to play an important role in the search for models beyond.

The discovery of exact asymptotic safety for non-Abelian gauge theories with matter [3–5] has raised substantial interest. Asymptotic safety explains how theories can be fundamental, predictive, and *interacting* at highest energies [6]. Initially put forward as a scenario to quantize gravity [7–10], asymptotic safety also arises in many other theories [11–14]. In particle physics, asymptotic safety offers intriguing new directions to ultraviolet (UV) complete the standard model beyond the confines of asymptotic freedom [15–17].

In this Letter, we investigate whether asymptotic safety can be achieved in supersymmetric gauge theories. In the language of the renormalization group (RG), asymptotic safety corresponds to an interacting UV fixed point for the running couplings [6]. Supersymmetry modifies fixed points and the evolution of couplings, because it links bosonic with fermionic degrees of freedom [4,18,19]. Additional constraints arise as bounds on the superconformal R charges [20] from both unitarity [21] and the a theorem [22–25]. Hence, our task consists of finding supersymmetric gauge theories without asymptotic freedom, but with viable interacting UV fixed points, and in accord with all constraints.

One arena in which we may hope to find reliable answers is that of perturbation theory. For sufficiently small couplings [26], the loop expansion and weakly interacting fixed points are trustworthy [4]. In this spirit, we obtain fixed points, phase diagrams, superconformal R charges, and UV-IR connecting trajectories for supersymmetric gauge theories in a controlled setting. Previously, this philosophy has been used successfully for proofs of asymptotic safety in nonsupersymmetric simple [3] and semisimple [5] gauge theories.

The model.—We consider a family of massless supersymmetric Yang-Mills theories in four space-time dimensions with product gauge group $SU(N_1)\otimes SU(N_2)$, coupled to chiral superfields (ψ, χ, Ψ, Q) with flavor multiplicities (N_F, N_F, 1, N_Q). The main novelty is the use of a semisimple gauge group, as otherwise asymptotic safety cannot arise at weak coupling [4,18]. For each superfield, we introduce a left- and right-handed copy with gauge charges as in Table I to ensure the absence of gauge anomalies. Also, viable models with asymptotic safety must have Yukawa couplings [4]. Therefore, we allow for superpotentials of the form

$$W = y\mathrm{Tr}[\psi_L\Psi_L\chi_L + \psi_R\Psi_R\chi_R], \tag{1}$$

where the trace sums over flavor and gauge indices. The superfields Q are not furnished with Yukawa interactions. The theory has a global $SU(N_F)_L\otimes SU(N_F)_R\otimes SU(N_Q)_L\otimes SU(N_Q)_R$ flavor and a $U(1)_R$ symmetry. Moreover, the theory is renormalizable in perturbation theory and characterized by two gauge couplings g_1 and g_2 and the Yukawa coupling y, which we write as

$$\alpha_1 = \frac{N_1 g_1^2}{(4\pi)^2}, \qquad \alpha_2 = \frac{N_2 g_2^2}{(4\pi)^2}, \qquad \alpha_y = \frac{N_1 y^2}{(4\pi)^2}. \tag{2}$$

Sending field multiplicities (N_1, N_2, N_F, N_Q) to infinity while keeping their ratios fixed reduces the number of free parameters down to three, which we choose to be

TABLE I. Chiral superfields and their gauge charges.

Chiral superfields	ψ_L	ψ_R	Ψ_L	Ψ_R	χ_L	χ_R	Q_L	Q_R
$SU(N_1)$	$\bar{\square}$	$\square$	$\square$	$\bar{\square}$	1	1	1	1
$SU(N_2)$	1	1	$\square$	$\bar{\square}$	$\bar{\square}$	$\square$	$\bar{\square}$	$\square$

$$R = \frac{N_2}{N_1}, \qquad P = \frac{N_1}{N_2}\frac{N_Q + N_1 + N_F - 3N_2}{N_F + N_2 - 3N_1},$$
$$\epsilon = \frac{N_F + N_2 - 3N_1}{N_1} \tag{3}$$

In the large-N limit [26], the model parameters (R, P, ϵ) are continuous. We can always arrange to find (3) with

$$1 < R < 3, \qquad P = \text{finite}, \qquad 0 < |\epsilon| \ll 1. \tag{4}$$

The smallness of ϵ ensures perturbative control in both gauge sectors [4,5], which is the regime of interest for the rest of this work (the general case is discussed elsewhere [27]). This completes the definition of our models.

Superconformal fixed points.—The running of couplings is controlled by the beta functions $\beta_i = d\alpha_i/d\ln\mu$, with μ denoting the RG momentum scale. To find accurate fixed points, we must minimally retain terms up to two loops in the gauge and one loop in the Yukawa beta functions [4]. Using the results of Refs. [28,29] and suppressing subleading terms in ϵ, we find

$$\beta_1 = 2\alpha_1^2[\epsilon + 6\alpha_1 + 2R\alpha_2 - 4R(3-R)\alpha_y],$$
$$\beta_2 = 2\alpha_2^2\left[P\epsilon + 6\alpha_2 + \frac{2}{R}\alpha_1 - \frac{4}{R}(3-R)\alpha_y\right],$$
$$\beta_y = 4\alpha_y[2\alpha_y - \alpha_1 - \alpha_2]. \tag{5}$$

Anomalous dimensions of the superfields are given by

$$\gamma_\Psi = (3-R)\alpha_y - \alpha_1 - \alpha_2,$$
$$\gamma_\psi = R\alpha_y - \alpha_1,$$
$$\gamma_\chi = \alpha_y - \alpha_2,$$
$$\gamma_Q = -\alpha_2. \tag{6}$$

up to corrections of the order of $\mathcal{O}(\epsilon\alpha, \alpha^2)$. The simultaneous vanishing of (5) implies fixed points and scale invariance. Besides the free Gaussian (G), the model has weakly coupled fixed points α^* of order ϵ. These are either of the Banks-Zaks (BZ) or gauge-Yukawa (GY) type, depending on whether the Yukawa coupling is free or interacting [4]. We find partially interacting Banks-Zaks (BZ_1, BZ_2) and gauge-Yukawa (GY_1, GY_2) fixed points and fully interacting ones (BZ_{12}, GY_{12}), all summarized in Table II. Results are exact to the leading order in ϵ, with higher loop orders only correcting subleading terms. We also note that (5), (6), and fixed points are universal and RG scheme independent at weak coupling [3,4].

At superconformal fixed points, our models display a global and anomaly-free $U(1)_R$ symmetry. In terms of the superfield anomalous dimensions (6), the R charges (not to be confused with the parameter R) read

$$R_i = 2(1 + \gamma_i^*)/3. \tag{7}$$

TABLE II. The G and all BZ and GY fixed points to leading order in ϵ.

Fixed point	G	BZ_1	BZ_2	GY_1	GY_2	BZ_{12}	GY_{12}
α_1^*	0	$-\frac{\epsilon}{6}$	0	$-\frac{\epsilon}{2(3-3R+R^2)}$	0	$\frac{PR-3}{16}\epsilon$	$\frac{3-4R-2PR^2+PR^3}{(R-1)(9-8R+3R^2)}\frac{\epsilon}{2}$
α_2^*	0	0	$-\frac{P\epsilon}{6}$	0	$\frac{-PR}{4R-3}\frac{\epsilon}{2}$	$\frac{1-3PR}{16R}\epsilon$	$\frac{R-2-3PR+3PR^2-PR^3}{(R-1)(9-8R+3R^2)}\frac{\epsilon}{2}$
α_y^*	0	0	0	$\frac{1}{2}\alpha_1^*$	$\frac{1}{2}\alpha_2^*$	0	$\frac{1}{2}(\alpha_1^* + \alpha_2^*)$

Nonperturbative expressions for the R charges are found using the method of a maximization [20]. For small couplings, findings agree with (6) and (7) and deviate mildly from Gaussian values, in accord with unitarity [21].

Asymptotic freedom of (5) is guaranteed for $P > 0 > \epsilon$. Then, all three couplings (2) are marginally relevant at the Gaussian UV fixed point. The set of asymptotically free trajectories is characterized by three free parameters, the initial values $0 < \delta\alpha_i(\Lambda) \ll 1$ at the high scale Λ. Some or all interacting fixed points of Table II arise within specific parameter ranges (3) and take the role of IR fixed points. Trajectories either run towards a regime with strong coupling and confinement or terminate at a superconformal IR fixed point. By and large, this is very similar to the generic behavior of asymptotically free nonsupersymmetric gauge theories [5].

Asymptotic safety.—Next, we turn to regimes (3) where asymptotic freedom is lost, starting with

$$P < 0 < \epsilon. \tag{8}$$

Clearly, the Gaussian has ceased to be the UV fixed point for the full theory, and one might wonder whether its role is taken over by one of the interacting fixed points in Table II. Available candidates in the regime (8) are BZ_2, GY_2, and GY_{12}. At the partially interacting BZ_2, only the Yukawa term (1) is a relevant perturbation. The theory becomes interacting in α_2 and α_y, yet α_1 remains switched off at all scales. From the eigenvalue spectrum, we learn that GY_{12}, once it exists, is IR attractive in all couplings. Hence, neither the Gaussian, nor BZ_2, nor GY_{12} qualify as UV fixed points. A new effect occurs at GY_2. While α_2 and α_y are irrelevant in its vicinity [4], the relevancy of α_1 now depends on the magnitude of α_2^* and α_y^* at GY_2. We find

$$\beta_1|_{GY_2} = -B_{1,\text{eff}}\alpha_1^2 + \mathcal{O}(\alpha_1^3),$$
$$B_{1,\text{eff}} = -2\epsilon + 2\epsilon P/Q_1, \tag{9}$$

with $Q_1(R) = (4R-3)/(R^3 - 2R^2)$. The first term in $B_{1,\text{eff}}$ is the conventional one-loop coefficient. It is negative in the regime (8) and documents the irrelevancy of α_1 at the Gaussian. The second term is sourced through the fixed point GY_2. Most notably, the sign of $B_{1,\text{eff}}$ is positive provided that

$$P < Q_1 < 0, \qquad 1 < R < 2, \qquad \epsilon > 0, \qquad (10)$$

thereby turning α_1 into a relevant coupling. We emphasize that the Yukawa term (1) is crucial to achieve $B_{1,\mathrm{eff}} > 0$; without it, the required change of sign would be impossible [4]. In other words, while α_1 is IR free close to the Gaussian or BZ_2 fixed points, it has become UV free close to the GY_2 fixed point. It is precisely for this reason that the gauge-Yukawa fixed point GY_2 takes the role of an asymptotically safe UV fixed point with one marginally relevant and two irrelevant directions.

The same mechanism is operative once $P, \epsilon < 0$, where α_1 and α_2 have interchanged their roles. Near GY_1, the effective one-loop coefficient for α_2 reads $B_{2,\mathrm{eff}} = 2(Q_2 - P)\epsilon$, with $Q_2 = (R-2)/(R^3 - 3R^2 + 3R)$. Consequently, α_2 becomes a relevant coupling for

$$Q_2 < P < 0, \qquad 1 < R < 2, \qquad \epsilon < 0, \qquad (11)$$

thereby promoting GY_1 to an UV fixed point. As soon as both gauge sectors are destabilized ($P, \epsilon > 0$), no fixed point other than the IR attractive Gaussian can arise. Theories are UV incomplete and must be viewed as effective. Figure 1 summarizes our results once $P < 0$, also indicating the parameter regions (10) and (11) with exact asymptotic safety.

From the UV to the IR.—At either of the superconformal UV fixed points, the elementary "quarks" and "gluons" are unconfined and appear as interacting (free) massless particles in one (the other) gauge sector. The free gauge sector acts as a marginally relevant perturbation which drives the theory away from the UV fixed point. The corresponding phase diagram in the regime (10) is shown in Fig. 2. It confirms that GY_2, unlike the Gaussian, is the unique UV fixed point. Close to the UV fixed point, the critical surface of asymptotically safe trajectories running out of it is given by

$$\alpha_1(\mu) = \frac{\delta\alpha_1(\Lambda)}{1 + B_{1,\mathrm{eff}}\delta\alpha_1(\Lambda)\ln(\mu/\Lambda)},$$
$$\alpha_2(\mu) = \alpha_2^* + \frac{2-R}{4R-3}\alpha_1(\mu),$$
$$\alpha_y(\mu) = \alpha_y^* + \frac{3R-1}{8R-6}\alpha_1(\mu). \qquad (12)$$

We emphasize that the theory has only one free parameter $\delta\alpha_1(\Lambda) \ll 1$ related to the relevant gauge coupling at the high scale Λ. Both α_2 and α_y have become irrelevant couplings and are strictly determined by α_1. [Similar expressions are found for the regime (11).] Dimensional transmutation leads to the RG invariant mass scale

$$\mu_{\mathrm{tr}} = \Lambda \exp[-B_{1,\mathrm{eff}}\delta\alpha_1(\Lambda)]^{-1}, \qquad (13)$$

which is independent of the high scale. It characterizes the scale where couplings stop being controlled by the UV fixed point. For RG scales $\mu \ll \mu_{\mathrm{tr}}$, we observe a crossover into another superconformal fixed point (GY_{12}) governing the IR. There, the elementary quarks and gluons of either gauge sector remain unconfined and appear as interacting massless particles, different from those observed in the UV.

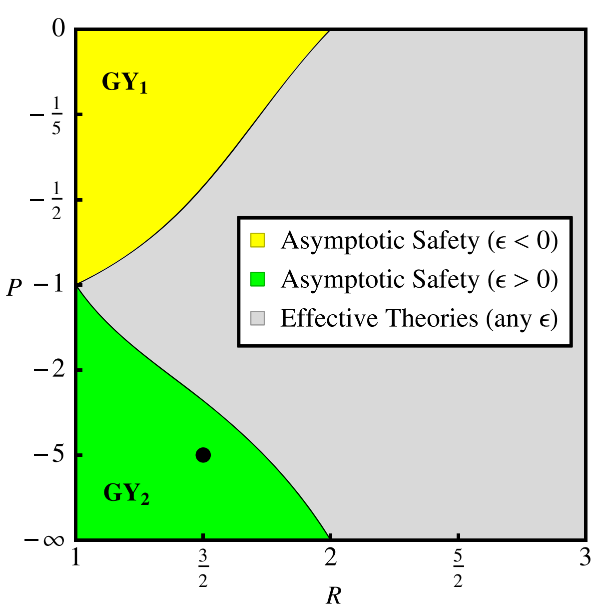

FIG. 1. Phase space for asymptotic safety, showing the parameter regions (10) and (11). Models in the gray-shaded area are UV incomplete. The P axis is scaled as $P/(1-P)$ for better display. The full dot indicates the example in Figs. 2 and 3.

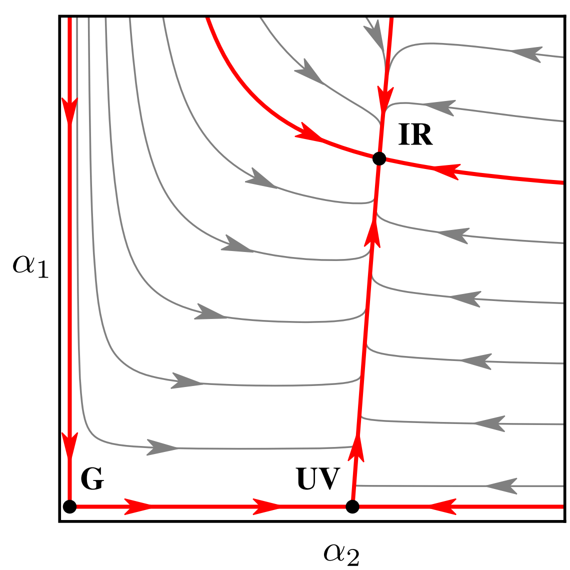

FIG. 2. Phase diagram with asymptotic safety for supersymmetry ($P = -5$, $R = (3/2)$, $\epsilon = (1/1000)$; Fig. 1) projected onto $\alpha_y = (\alpha_1 + \alpha_2)/2$. Trajectories are pointing towards the IR. Notice that α_1 is destabilized and asymptotic freedom is absent. Dots show the Gaussian, the UV, and the IR fixed points. Also shown are separatrices (red) and sample trajectories (gray).

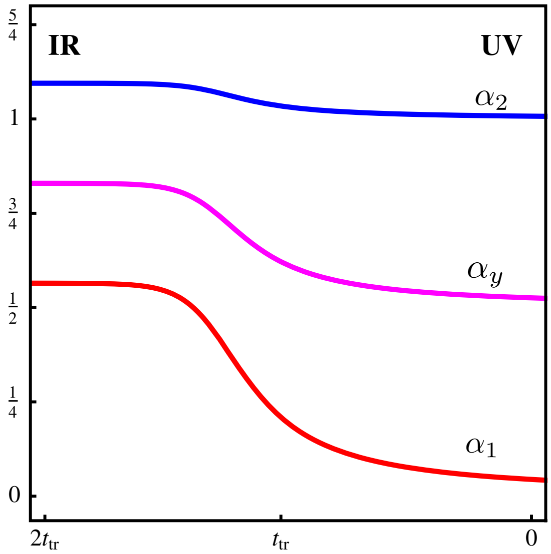

FIG. 3. The running couplings $\alpha_i(t)$ in units of RG time $t = \ln(\mu/\Lambda)$ along the separatrix from the UV to the IR fixed point. Parameters are as in Fig. 2. All couplings in units of $\alpha^*_{2,\mathrm{UV}}$ with $t_{\mathrm{tr}} = \ln(\mu_{\mathrm{tr}}/\Lambda)$ and Λ the high scale; see (13).

Figure 3 exemplifies the running of couplings from the UV to the IR.

The UV fixed point persists in the presence of mass terms for the chiral superfields. Once masses are switched on, with or without soft supersymmetry-breaking ones such as those for the "gluinos," they lead to decoupling [30] and low-energy modifications of the RG flow (5). Then, UV safe trajectories may terminate in regimes with strong coupling and confinement in the IR, with or without softly broken supersymmetry.

Asymptotic safety and the a theorem.—We are now in a position to establish consistency with a more formal aspect of the renormalization group known as the a theorem [22–25]. It states that the central charge $a = (3/32)\{2d_G + \sum_i (1-R_i)[1-3(1-R_i)^2]\}$ [24] must be a decreasing function along RG trajectories in any $4d$ quantum field theory (d_G denotes the dimension of the gauge groups and i runs over all chiral superfields). Using (6), (7), and Table II, we find

$$\Delta a \equiv a_{\mathrm{UV}} - a_{\mathrm{IR}} > 0 \qquad (14)$$

on any of the UV-IR connecting trajectories in the parameter ranges (10) and (11) shown in Fig. 1. Had the IR limit been the Gaussian, validity of the a theorem implies strong coupling and large R charges in the UV, at least for some of the fields [18,24]. In our models, this implication is circumvented, because the IR is not free. In fact, there is not a single trajectory flowing from the UV fixed point to the Gaussian (Fig. 2), which again is in accord with the a theorem ($a_{\mathrm{UV}} - a_G < 0$).

Discussion.—In supersymmetry, and for superpotentials of the form (1) including mass terms, the scalar potential is always a sum of squares of absolute values [31]. Hence, the stability of the quantum vacuum is automatic. Also, a fixed point for the gauge and Yukawa couplings implies a fixed point for the scalar potential. Without supersymmetry, the physicality of scalar fixed points and vacuum stability do not come by default [4] and must be checked case by case [5,32].

Also, without supersymmetry, at least one Yukawa coupling is required to help generate an interacting UV fixed point [4]. Invariably, this reduces the number of fundamentally free parameters in the UV by at least one, thereby enhancing the predictive power [3]. In supersymmetry, asymptotic safety at weak coupling cannot arise with only a single gauge factor [4,18]. Then, as we have seen in (12), at least one of the Yukawa couplings together with at least one of multiple gauge couplings must be nontrivial in the UV, thereby reducing the number of free parameters by two. We conclude that supersymmetry additionally enhances the predictive power of asymptotic safety.

We have shown that asymptotic safety is operative in supersymmetric gauge theories. Yukawa couplings continue to play a distinctive role at weak coupling, as they do for asymptotic safety without supersymmetry [4]. Explicit examples with superpotential (1) and matter content as in Table I are provided, including the phase space (Fig. 1) and phase diagram (Fig. 2). Results are consistent with unitarity and the a theorem. Our construction makes it clear that asymptotic safety exists in supersymmetry beyond the models discussed here. It is interesting to include more gauge groups, expand Yukawa sectors, switch on mass terms, and explore the potential for asymptotically safe supersymmetric model building.

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